Bubble formation in gelatin: A model for decompression sickness*

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Decompression sickness follows a reduction in ambient pressure and is a result of bubble formation in blood or tissue. A striking feature of the syndrome is that almost any body part, organ, or fluid can be affected, including bone. This generality suggests a common basis in the physical and chemical properties of water, particularly those relating to cavitation. We report here a systematic study of some of these properties, carried out with transparent gelatin—an aqueous medium that is especially convenient for this purpose since it yields bubbles that are stationary and can be counted and measured.

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I. INTRODUCTION

Many of the principles used in calculating modern decompression tables for divers and compressed-air workers are merely assumptions, as yet unverified by experiment. A few are probably wrong. Fortunately, the main use of such principles has been to provide a loose framework for correlating diving experience. The result, after 100 yr of large-scale compressed-air work, is that modern diving tables are essentially empirical and relatively safe. We believe, however, that further significant improvements may be possible through a better understanding of the basic processes of gas transport, bubble formation, bubble growth, and bubble elimination in tissue. To this end, we have carried out a series of experiments in which samples of transparent gelatin are pressurized and then decompressed, resulting in bubble formation. Certain highlights of this program have already been reported, 1-3 but the major portion of the data is presented and interpreted here.

The principal experimental findings are that (i) pre-existing gas nuclei account for at least 99.9% of the bubbles formed by decompressing gelatin, (ii) the great majority of these nuclei are already present in the water before the gelatin is prepared, (iii) the Haldane-ratio principle is not applicable to bubble formation in gelatin, (iv) to a first approximation, the number of bubbles formed in gelatin is a function only of the supersaturation pressure for a given gas and underlying radial distribution of gas nuclei, (v) in contrast with the principle of zero-supersaturation decompression, these underlying distributions often permit relatively large supersaturation pressure to be withstood without bubble formation, (vi) gas nuclei can be crushed, and the subsequent number of bubbles greatly reduced by rapid compression. A further result of this investigation is the mathematical derivation of an optimum decompression schedule for gelatin that is faster and yields fewer and smaller bubbles in gelatin than does the corresponding U.S. Navy schedule.

The experimental apparatus is described in Sec. II. The test compression–decompression schedules and resulting data are analyzed and compared with the predictions of cavitation theory in Sec. III. An optimum decompression schedule for gelatin is derived in Sec. IV, and the possible relevance of this work to humans is discussed in Sec. V.

II. APPARATUS

The pressure vessel used in this experiment is shown in Fig. 1. The maximum permissible gas pressure is 300 psig with a safety factor of 5. The vessel is cylindrical and has an inside diameter of 15 cm and a length of 27 cm. A plexiglass window at one end of the vessel permits the counting chambers containing the gelatin samples to be viewed and photographed either directly or through a microscope at any time during a pressure schedule. Visibility is enhanced by small lights mounted just outside the window. Initially, the vessel contains air at ambient temperature and pressure, and nitrogen of 99.9% purity is added as required. The counting chambers containing the gelatin samples are immersed in a water bath at 21°C to insulate the samples from changes in gas temperature that occur during compression and decompression.

As viewed through the plexiglass window, the counting chambers are 27 mm wide and 6 mm along the line of sight. The base of each successive chamber is lower than the previous one to permit simultaneous viewing of...
all of the chambers from the front window. Unless otherwise noted, each chamber was filled with gelatin to a depth of 4 mm. A grid scored on the external face near the bottom of each glass chamber provides a reference for filling and for counting bubbles. Originally there were four counting chambers, but one chamber was broken during the course of the experiment, and subsequent tests were carried out with only three.

When gelatin was newly mixed prior to each test schedule, the variability of the results was intolerable. To improve the reproducibility, a single large batch of gelatin was mixed and piped onto 400 individual 10-ml aliquots. These were stored at about -20°C in a deep freeze. Unless otherwise noted, all of the data reported here were obtained from this one batch. Within the purely statistical errors calculated from the square root of the number of bubbles counted, samples taken from this batch yielded results that were consistent and reproducible over a period exceeding 1 yr.

The standard batch was prepared by adding 127 g of Knox unflavored gelatin crystals to 5 l of distilled water that had stood overnight. After being mixed, the gelatin solution was maintained at 39°C for 50 min, during which time it was occasionally stirred gently. It was then piped into the aliquots and frozen, as previously indicated. Prior to each test, the aliquot bottle containing a frozen sample was placed in a water bath at 40°C for 20 min, and the resulting sol was then pipetted into the glass counting chambers.

Pressurization was performed with gelatin in either the gel or the sol state. In most cases, two gel and two sol samples were run simultaneously. Samples initially in the sol state remained so through the compression stage and became a gel during the several hours spent at high pressure. Other samples were converted to gel prior to pressurization by immersing the respective counting chambers in ice water for 10 min. No bubbles were visible before decompression in either case. Furthermore, gel and sol samples yielded the same number of bubbles, within the calculated statistical errors, for all pressure schedules tried. This implies that bubble formation is not influenced by any trapping action of the gel and that the elastic shearing forces, which dominate bubble growth in elastomers, are negligible in comparison with the surface tension. This is expected since the gelatin mixture used here is exceptionally weak and barely sufficient to permit gelatin at 21°C.

Figure 2 is a photograph of a counting chamber and gelatin sample taken a few minutes after decompression. Ordinarily, bubbles became visible within seconds after a rapid decompression and grew for some minutes before stabilizing. If no further changes in pressure occurred, the sample would remain stable for several days. This photograph is typical in that bubbles were approximately spherical and generally well separated from one another. Bubbles in the lower 3 mm of the counting chamber were of fairly uniform radius and distribution in space, while those in the upper 1 mm and meniscus were noticeably smaller and appeared to be more numerous than elsewhere. Only bubbles in the lower 3 mm, defined as the fiducial volume, were counted.

The magnification of the microscope was 7, permitting bubbles to be observed down to about 10 μ in radius. This was not a serious limitation as far as bubble counting was concerned since virtually no bubbles occurred with radii near this observational threshold. The tendency of bubbles to form with fairly large and uniform radii in a given test has been noted by other workers and is an important observation. It will be clear shortly that the number of bubbles is equal to the number of gas nuclei larger than the threshold for bubble formation following decompression, whereas bubble size is determined by how much gas diffuses into each nucleus. In effect, each supercritical nucleus collects all of the gas dissolved within its immediate neighborhood.

III. DATA

A. Pressure schedules

A generalized pressure schedule is shown in Fig. 3. The pressure at which the gelatin is mixed is \( p_0 \), the final pressure is \( p_f \), and the maximum pressure is \( p_m \). Some minutes \( t_m \) after the maximum pressure \( p_m \) is
reached, the pressure is lowered and held for several
hours ($t_p$) at $p_s$ until saturation of the sample by the sur­
rounding gas is essentially complete. The supersatura­
tion pressure $p_{sa}$ is defined to be the difference in the
gas tension $\tau$ in the sample and the ambient pressure
$p_{amb}$. For complete saturation followed by a rapid de­
compression, this gives

$$ p_{sa} = \tau - p_{amb} = p_s - p_f. $$

(1)

**B. Temperature**

The nominal temperature at which experiments were
carried out was 21°C. Typical temperatures measured
in the water bath prior to decompression were slightly
higher and fluctuated by less than one centigrade degree.
To verify that these fluctuations did not affect the bubble
counts, 12 separate runs were made over the range
from 21.2 to 24.9°C. Within statistical errors of ±13% at
each setting, no systematic differences were ob­served. The range of this test is quite narrow because the
gel changed to sol at about 25°C.

**C. Saturation times**

The solution of the diffusion equation for an infinite
plane of uniform thickness is given by Crank. Using
his results and the measured diffusion constant,

$$ D = 1.1 \times 10^{-4} \text{ cm}^2/\text{min}, $$

(2)

for nitrogen in a gel of 1–2% agar in water, we obtain
theoretical time constants at the bottom and top of the
fiducial volume of, respectively,

$$ \sigma(0 \text{ mm}/4 \text{ mm}) = 74 \text{ min}, $$

(3a)

$$ \sigma(3 \text{ mm}/4 \text{ mm}) = 19 \text{ min}. $$

(3b)

These limits may be compared with the time constants,
$\sigma_1 = 1 \text{ min}$ and $\sigma_2 = 128 \text{ min}$, obtained by Dieter$^5$ in fitting the U.S. Navy Standard Air Decompression Tables of
1958$^8$ with a model incorporating two tissue types. The
British decompression procedures described in Ref. 10
are based on a single 75-min tissue.

The time constants for diffusion were investigated experimen­tally by varying the time $t_s$ at saturation
pressure $p_s$ in a schedule similar to that in Fig. 3 with
$p_f = p_0 = 0 \text{ psig}$ and $p_{m} = p_s = 73 \text{ psig}$. The results were
consistent with the above calculations and indicate that saturation was essentially complete after a total time
at high pressure of $t_m + t_s = 5.25 \text{ h}.$

**D. Denucleation by centrifuging gelatin**

Several methods of denucleation$^{11}$ were compared by
subjecting gelatin samples to the pressure schedule
shown in Fig. 3 with $p_f = p_0 = 0 \text{ psig}$ and with $p_m = p_s = 300
\text{ psig}$ for a total saturation time of $t_m + t_s = 5.25 \text{ h}.$ Ord­
inaril such a schedule results in about 400 macro­
scopic bubbles per sample fiducial volume. Centrifuging
was carried out by (i) placing part of the contents of an
aliquot at the bottom of a centrifuge tube and allowing it
to gel, (ii) filling the remaining volume of the tube with
distilled water, and (iii) spinning the tube for 15 min at
20000 rpm. This produces a hydrostatic pressure of
about 100 atm at the top of the gelatin mass. The sub­
sequent yield was $(0.4 \pm 0.15)$ bubbles per sample,
averaged over 18 samples. This corresponds to $(0.10
\pm 0.04)\%$ of the number obtained with centrifuging. The
fact that centrifuging is effective in the gel state sug­
gests that centrifuging eliminates nuclei by compression or crushing rather than by precipitation. Crushing is a
specific test for gas nuclei. Thus, gas nuclei accoun for
at least 99.9% of the bubbles formed in gelatin.

**E. Denucleation by centrifuging water**

To determine whether gas nuclei are associated with
powdered gelatin, with the mixing and preparation of
samples, or with water per se, we prepared a special
batch of gelatin using distilled water that had been
centrifuged at 20000 rpm for 15 min. The standard propor­tions of powdered gelatin to water were used. When
subjected to the test schedule described in Sec.
III D, this batch yielded 28 ± 3 bubbles per sample
averaged over three samples. The exact bubble count
probably depends on the test batch and schedule. Neve­
ertheless, this result indicates that approximately 93% of the nuclei are already present in the water with which
the gelatin is prepared.

**F. Seeding with gelatin powder**

To determine the source of the remaining nuclei, i.e.,
those not associated with water in the previous
test, gelatin powder was placed at the bottom of indi­
vidual counting chambers in variable quantities. De­
nucleated sol was then added and the sample allowed to
stand until the powder dissolved. The pressure schedule
described in Sec. III D yielded no bubbles for a sample
into which no powder had been introduced and 7, 0.4 ± 2.6
bubbles for a sample seeded with 4 mg of gelatin pow­
er. A third sample seeded with 8 mg of powder yielded
$15 ± 4$ bubbles. We conclude that some nuclei are asso­
ciated with the gelatin powder and that the number of
such nuclei is roughly proportional to the powder used.
An estimate of the fraction of the nuclei introduced into
the standard batch via the original gelatin powder can be
made from these data and is consistent with the 7% not
associated with water. Furthermore, we have found no
evidence that any of the procedures used in preparing
the standard batch (e.g., stirring, heating, cooling,
freezing, melting, etc.) does introduce nuclei. We con­
clude that about 93% of the nuclei observed in gelatin
are associated with the water and the remaining $7\%$ with
the gelatin powder.

**G. Experimental crushing times**

The time required to crush gas nuclei was investi­
gated experimentally by subjecting samples to a pres­
sure spike at the beginning of a pressure schedule and
by varying the period $t_m$ during which the pressure was
held at the maximum setting $p_m$. Specifically, a sche­
dule similar to that shown in Fig. 3 was followed with
$p_f = 0 \text{ psig}$, $p_m = 300 \text{ psig}$ during a variable time $t_m$, $p_s
= 150 \text{ psig}$ during $t_s = 5.25 \text{ h} - t_m$, and $p_f = 0 \text{ psig}$. The
combined results for two sol and four gel samples per
data point are plotted versus $t_m$ in Fig. 4. Three hori­
zontal scales are shown, one in sec and the other two in
min, to permit the details of the short-term and long­
term behavior to be clearly seen on the same graph.
The two scales in minutes give $t_m$ and $t_s$, respectively.
The effects of compression and decompression have been
studied to understand the behavior of gas nuclei within a gelatin
sample. The most important feature evident in Fig. 4 is that
the effects of compression and decompression have been
cleanly separated. More than 75% of the bubbles that
would ordinarily form in a standard batch of gelatin were crushed
during compression and the first second at
saturation pressure is produced. At
the shortness of this time underscores the fact that crushing occurs because of the
rapid increase in hydrostatic pressure, rather than because of the relatively slow rise in
gas tension within the sample. The remaining 25% of the bubbles are associated with (i) gas nuclei that require several
minutes to be crushed and (ii) gas or other nuclei that resist crushing at this level. After
the number of bubbles has been reduced by 97%, and the crushing phase is complete. This is followed by a broad interval over which the bubble count is independent of
the early part of this interval is characterized by a rising gas tension \( \tau \) in the gelatin that remains below \( \rho_s = 150 \text{ psig} \). In the central region, \( \tau \) begins to exceed \( \rho_s \) but returns to \( \rho_s \) before the final decompression takes place. At \( t_s = 240 \text{ min} \), the saturation time \( t_s \)
= 75 min is not quite sufficient to allow \( \tau \) to decrease to \( \rho_s \) before decompression, so that a larger final super­
saturation pressure is produced. At \( t_s = 310 \text{ min} \), \( t_s \)
= 5 min is so short that samples are essentially at \( \tau = \rho_s = 300 \text{ psig} \) just before the final decompression occurs. The rate of rise in the bubble count on the right-hand side of Fig. 4 is indicative of the diffusion time constants and is consistent with the calculated range of 19—74 min in Eqs. 3(a) and 3(b).

H. Critical radius

The equilibrium radius for a gas or vapor phase sur­
rounded by a liquid is given by the well-known equation\(^{15,16} \)

\[
\rho = \frac{2\gamma}{(\rho_s - \rho_{amb})},
\]

where \( \gamma \) is the surface tension, \( \rho_s \) is the pressure inside the gas phase, and \( \rho_{amb} \) is the ambient hydrostatic pressure. A gas phase satisfying this equation is in instantaneous mechanical equilibrium but will expand or contract by diffusion according to the equation\(^{15,16} \)

\[
\frac{d\rho}{dt} = \frac{2\gamma}{\rho_{amb} - \rho_s} - \frac{2\gamma}{\rho_s}
\]

Equation (5) leads to the notion of critical radius, defined by

\[
\rho_c = \frac{2\gamma}{(\rho_s - \rho_{amb})}.
\]

Gas phases characterized by radii larger than \( \rho_c \) ordinarily expand by diffusion, while those with radii smaller than \( \rho_c \) contract.

The above equations can be applied to the decom­
pression of saturated gelatin as follows. Immediately prior to a rapid decompression, the sample is at equilibrium at the saturation pressure \( \rho_s \) (i.e., \( \tau = \rho_s \)). Immediately after a rapid decompression from \( \rho_s \) to \( \rho_s \), the ambient pressure is \( \rho_f \) \( (\rho_{amb} = \rho_f) \). The critical radius and supersaturation pressure are then related by

\[
\rho_s = \frac{2\gamma}{(\rho_s - \rho_f)} = \frac{2\gamma}{\rho_{sat}},
\]

(7a)

\[
\rho_{sat} = \rho_s - \rho_f = 2\gamma/\rho_c.
\]

(7b)

To every supersaturation pressure \( \rho_{sat} \) there corre­
sponds a critical radius \( \rho_c \) such that any gas phase (or gas nucleus) in the sample with characteristic radius larger than \( \rho_c \) following decompression will grow into a macroscopic bubble.

I. Range of nuclear radii

The smallest nuclear radius that could be probed in this experiment was determined by the surface tension of the gelatin and the 300-psig rating of the pressure vessel. We have measured the surface tension of the standard gelatin batch by observing the contact angle of the sol in a capillary tube.\(^{12} \) The result is

\[
\gamma = (51 \pm 5) \text{ dyn/cm},
\]

(8)

which may be compared with a value of about 72 dyn/ cm for water at room temperature.\(^{15} \) The agreement of the bubble counts indicates that the surface tension is effectively the same for gel as for sol. Substitution of Eq. (8) into Eq. (7a) yields

\[
\rho_c = (14.8 \pm 1.5) \mu \text{ psi}/(\rho_s - \rho_f).
\]

(9)

Setting \( \rho_s - \rho_f \) equal to 300 psi gives

\[
\rho_c = (0.0490 \pm 0.003) \mu.
\]

(10)

In principle, any radius larger than \( \rho_{sat} \) could have been detected. It is strictly an experimental question what the characteristic radii were for the gas nuclei present following decompression. To determine the largest critical radius that could be produced in the standard batch, two gel samples were decompressed...
serially in a vacuum chamber, beginning at atmospheric pressure and continuing until bubbles were first seen. Evacuation was used in this test to ensure that no crushing of gas nuclei took place. An average pressure change of $p_f - p_a = -12$ psi ($p_f = 2.7$ psia) was required, corresponding via Eq. (9) to a nuclear radius of 

$\gamma_{\text{max}} = (1.24 \pm 0.13) \mu$. (11)

### J. Mixing at pressure

A further test of Eq. (7a) was obtained by mixing gelatin at $p_a = 100$ psig, using water that had stood 65 h at this pressure in a shallow container. Some crushing of the nuclei in water may have occurred, but those associated with the gelatin powder were expected to survive. The mixture was maintained in the sol state at 40°C until all visible bubbles had risen to the surface. Gelatin was then pipetted directly into three counting chambers, which were immersed in ice water to form the gel. As usual, the test schedule was carried out at slightly above 21°C. Decompression by 12 psi yielded no bubbles. However, a small increase in supersaturation pressure to 16 psi resulted in a substantial number, 14 ± 2 bubbles/sample, which extrapolates to a threshold near the 12-psi value obtained in the analogous test begun at 1 atm. We conclude that, for a given sample, surface tension, and underlying distribution of nuclear radii, the threshold for bubble formation depends only upon the supersaturation pressure.

### K. Haldane ratio

The Haldane-ratio principle, upon which the U.S. Navy and British Decompression Tables are partially based, states that it is the ratio of (tissue gas tension)/(ambient pressure) = $T/P_{\text{amb}} = p_f/p_a$, and not the difference $p_a = T - p_{\text{amb}} = p_f - p_{\text{amb}}$, that determines how rapidly a diver can surface safely. To compare these points of view, we have subjected gelatin samples to simple schedules for which $T/p_{\text{amb}}$ was varied while $p_{\text{amb}} = T - p_{\text{amb}}$ was held constant. To insure that gas nuclei were crushed to the same extent in each case, all samples were initially compressed to the same maximum pressure $p_a$ for 20 min. The pressure was then reduced to the desired saturation level $p_s = T$ for 5.25 h before being lowered to the final pressure $p_f = p_{\text{amb}}$.

Results for a typical pair of data sets having the same values of $p_m$ and $p_a - p_f$ ($p_m = 300$ psig, $p_a - p_f = 150$ psig) are given in Table I. The difference in Haldane ratios $p_f/p_{\text{amb}}$ is large (1.91 versus 11.2), yet there is very little difference in the averaged bubble counts. The third data set has a different value of $p_a$ ($p_m = 150$ psig) and gives about 25 times as many bubbles as the first two entries—In spite of the fact that $p_a - p_f$ is the same for all three entries and $p_f/p_{\text{amb}}$ is the same for the last two entries. The third data set illustrates again the importance of crushing, while the first two sets lead to the following conclusions: (i) that the Haldane ratio per se is not a significant factor in determining the number of bubbles that result from decompressing gelatin, and (ii) that to a first approximation, for a given sample, surface tension, and underlying distribution of nuclear radii, the number of bubbles formed in gelatin is a function only of the supersaturation pressure.

#### L. 1-atm envelope

Figure 5 is a plot of the number of bubbles per sample versus supersaturation pressure for a series of schedules having $p_f = p_a = 0$ psig and variable $p_m$. Such schedules, ending always at atmospheric pressure, yield the largest supersaturation pressure and

<table>
<thead>
<tr>
<th>$p_m$ (psig)</th>
<th>$p_a$ (psig)</th>
<th>$p_f$ (psig)</th>
<th>$p_f/p_{\text{amb}}$</th>
<th>$p_a - p_f$ (psig)</th>
<th>Samples</th>
<th>Bubbles</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>300</td>
<td>150</td>
<td>1.91</td>
<td>12</td>
<td>14 gel</td>
<td>14.6 ± 1.0</td>
</tr>
<tr>
<td>300</td>
<td>150</td>
<td>0</td>
<td>11.2</td>
<td>150</td>
<td>16 gel</td>
<td>16.9 ± 1.1</td>
</tr>
<tr>
<td>150</td>
<td>150</td>
<td>0</td>
<td>11.2</td>
<td>150</td>
<td>8 gel</td>
<td>20.4 ± 1.6</td>
</tr>
</tbody>
</table>

![Figure 5](image-url)
The number of bubbles obtained at each of these certain points on the 1-atm envelope with a compression rate of approximately 3 psi/sec decreased sharply with supersaturation pressure, beginning at a threshold near 15-20 psi and continuing until an apparent maximum is reached in the vicinity of 210 psi. 

To verify that the number of bubbles actually decreased as $p_m = p_s$ was raised from 210 to 300 psig, 6 gel and 3 sol samples were subsequently tested at each of these pressures. The ratio of the 300-psi data to the 210-psi data was 0.790 ± 0.017.

The decline observed in the number of bubbles versus supersaturation pressure in Fig. 5 results from crushing of gas nuclei. Somewhere in the vicinity of 15 psig data was supersaturation pressure in Fig. 5 results from compression. Decompression can cause new ones of smaller radius to grow into bubbles. Crushing was reduced by repeating certain points on the 1-atm envelope with a “slow compression” approximated by stepping the pressure by 15 psi every $\frac{1}{2}$ h. Slow compression permits some diffusion of gas into nuclei, allowing the nuclear surfaces to remain near their equilibrium configurations and thereby resist crushing. The number of bubbles obtained at the slow-compression points increases monotonically up to the maximum supersaturation pressure 300 psig obtainable in this experiment.

The data of Fig. 5 are replotted in Fig. 6 as a function of the critical radius $r_c$ calculated via Eq. (9). Estimates of the bubble numbers (divided by 100) for the slow compressions to 100, 210, and 300 psig, which were too large to be counted rigorously, are also shown. In principle, the ordinate gives the number of gas nuclei that had radii larger than the critical radius $r_c$ immediately following decomposition. The derivatives of the rapid and slow curves with respect to $r_c$ would then yield nuclear radial distributions, i.e., the number of gas nuclei versus radius, for the two cases. Because of crushing, however, and the fact that the crushing is different at each point, neither curve faithfully represents the integral radial distribution.

M. Loci of constant saturation pressure

The variation in the number of bubbles with supersaturation pressure $p_m = p_s - p_f$ was studied for constant crushing $p_m = p_s$ by taking data at fixed saturation pressure $p_s$ and variable final pressure $p_f$, with an initial pressure of $p_s = 0$ psig. In Fig. 7 the results are plotted for, respectively, 120, 210, and 300 psig. The one-atmosphere envelope is also shown for purposes of orientation, all compressions in this figure being rapid.

The data of Fig. 7 are replotted in Fig. 8 as a function of critical radius, calculated in the usual way. In

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**FIG. 6.** Data of Fig. 5 replotted as a function of the critical radius $r_c$. The number of bubbles is interpreted as being equal to the number of nuclei present in the original sample with a characteristic radius larger than $r_c$ following decompression.

**FIG. 7.** Rapid-compression constant-saturation-pressure curves defined by $p_s = 0$ psig, $p_m = p_s$, and variable $p_f$ and $p_m = p_s - p_f$. The 1-atm envelope for rapid compression is shown for purposes of orientation.
further insight into the crushing process, which seems to cut off. This seems to imply that the approximation expected to yield the same number of bubbles for a given total nuclei, is smallest in this case and most easily overcome either during compression or during decompression. It cannot be determined from these data alone whether the missing gas nuclei have been definitively extinguished or whether they have simply been reduced to a progressively smaller size. In either case, the process is irreversible in the sense that gas nuclei did not return to their initial condition during 5.25 h at the saturation pressure $p_s$.

The one-atmosphere envelope for slow compression and the slow-compression data for $p_s = 300$ psig, averaged over two gel and one sol samples, are also shown in Fig. 8. Slow compression in the later case consisted of 20 increments of 15 psig with a 2-h stop after each increment. The slow curve for $p_s = 300$ psig bears the same relation to the slow one-atmosphere envelope as the respective fast curves bear to one another, namely, the envelope in each case is rather sharply compressed or cut off. This seems to imply that the approximation of slow compression used here is still not slow enough and that gelatin mixed with saturated water at the saturation pressure would yield still higher bubble counts and reflect more faithfully the underlying radial distributions of the gas nuclei.

N. Zero supersaturation decompression

The zero supersaturation principle, proposed as an alternative to the Haldane-ratio principle, states that the inert gas tension $T$ should at no time exceed the ambient pressure $p_{amb}$, i.e.,

$$\tau < p_{amb}.$$  (12)

In animals breathing air, this condition can be satisfied during decompression by taking advantage of the inherent unsaturation that results from oxygen metabolism and the fact that CO$_2$ is more soluble in blood and tissue than O$_2$.

In gelatin, there is no inherent unsaturation. Furthermore, rather large supersaturation pressures can often be withstood without cavitation. This is evident in Fig. 7 and also in the centrifuging tests which yield "de-nucleated" samples capable of supporting at least 300 psi. We conclude that the zero supersaturation principle of decompression is not applicable to gelatin and that it can be violated safely, in some cases, by hundreds of psi.

IV. OPTIMUM DECOMPRESSION OF GELATIN

The criterion that must be satisfied if a physiologically insignificant number of bubbles shall form during decompression of gelatin is

$$p_{ss} = T - p_{amb} < 2\gamma/r_c^{max},$$  (13)

where $r_c^{max}$ is the maximum critical radius at which a significant number of nuclei are present. Zero supersaturation decompression, defined by Eq. (12), is a special case of Eq. (13) that is applicable when gas nuclei occur with very large radii.

For a saturated gelatin sample, the slowest "tissue type" is at the bottom and is characterized by a single time constant $\sigma^*$. The diffusion equation,

$$\frac{dT}{dt} = \frac{p_{amb}(t) - \tau(t)}{\sigma^*},$$  (14)

reduces to

$$\frac{dT}{dt} = -\frac{2\gamma}{\sigma^* r_c^{max}},$$  (15)

when $p_{amb} - \tau$ is obtained from Eq. (13), adopting the equals sign so that decompression is as rapid as possible, consistent with negligible bubble formation. Differentiating Eq. (13) then gives

$$\frac{dp_{amb}}{dt} = \frac{dT}{dt} = -\frac{2\gamma}{\sigma^* r_c^{max}}.$$  (16)

The optimum decompression schedule for a saturated gelatin sample is thus remarkably simple and consists of only two phases:

1) a fairly large first pull to

$$p_{amb}^* = \tau^* - (2\gamma/r_c^{max}),$$  (17)
where \( \tau^* \) is the gas tension immediately prior to decompression, and

2) a straight-line trajectory that begins at the end of the first pull and decreases with constant slope given by Eq. (16) until the final pressure \( p_f \) is reached.

The gas tension at the bottom of the sample follows another straight-line parallel to \( P_{\text{gas}}(t) \) and displaced above it by \( 2\gamma/\kappa_{\text{c}} \). After \( P_{\text{gas}}(t) \) reaches \( p_f \), the decay of the gas tension is exponential and \( \tau \) approaches \( p_f \) asymptotically.

In Fig. 9, a profile of the optimum form is compared with the U.S. Navy Decompression Schedule for a simulated 40-min dive to \( p_a = 100 \text{ ft of sea water} (= 44 \text{ psig} = 4 \text{ atm absolute}) \). For this test only, the counting chambers were filled to a depth of 2 mm instead of the usual 4 mm, and bubbles were counted only in the lowest 1 mm. The calculated time constants for this fiducial volume are 12.7 min at the top and 18.5 min at the bottom. The gas tension \( \tau \), plotted in Fig. 9 for a 15-min time constant, is nearly equal to \( p_a \) just prior to decompression.

The first ascent and slope of the optimum schedule were determined empirically in several trials. The resulting schedule was completed in 12 min and yielded \( 0.42 \pm 0.19 \) bubbles per sample averaged over 8 gel and 4 sol samples. The U.S. Navy schedule required 17 min and gave \( 12.9 \pm 1.0 \) bubbles per sample averaged over 8 gel and 4 sol samples. The U.S. Navy schedule proved to be unsafe for gelatin because the first ascent was too large, probing gas nuclei down to about 0.4 \( \mu \) in radius. A large first ascent is characteristic of tables based in part on the Haldane-ratio principle. The optimum schedule was less severe, and only those nuclei with radii larger than about 0.8 \( \mu \) grew to form macroscopic bubbles. The difference in bubble counts is, in fact, a measure of the number of gas nuclei per sample with radii in the range 0.4–0.8 \( \mu \) immediately following decompression.

V. DISCUSSION

The theoretical tensile strength of pure water exceeds 1000 atm, yet bubble formation is often observed in humans and in distilled water at supersaturation pressures below 1 atm. This difference cannot be explained by solid impurities with smooth surfaces. On the other hand, numerous experiments have shown that cavitation threshold for distilled water can be increased significantly by degassing or by a preliminary application of static pressure—a technique that is also highly effective for gelatin. These are specific tests for gas nuclei, and it is therefore evident that the precocious onset of cavitation in distilled water must be due mainly to the presence of such nuclei, even though their origins and the mechanisms stabilizing them are poorly understood.

It is reasonable to assume that preexisting gas nuclei also play a role in bubble formation that occurs in supersaturated humans. If so, there would be a number of physiological implications. For example, the acclimatization to decompression observed among caisson workers may result from the depletion of gas nuclei. Similarly, it may be possible to "denucleate" humans to some degree with pressure spikes, drugs, or other means. The Haldane-ratio principle would not apply to gas nuclei, and tables based in part on this idea would be expected to result in significant bubble formation on the large first ascent, as demonstrated in the gelatin model. That this actually happens is suggested by the observation of numerous macroscopic bubbles in humans, even during conventional asymptomatic decompressions. Since bubbles relieve the gas tension, and thus lower the rate at which gas diffuses out of bulk tissue, it follows that tables which avoid bubble formation would be not only safer but also faster than those in use today.

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*****************R. C. Weast, Handbook of Chemistry and Physics (Chemical Rubber Co., Cleveland, 1975).}